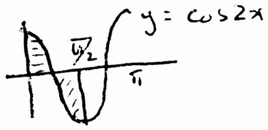
Exam Revision Answers

1. $f'(x) = -30(7-2x)^2$

$f'(4) = -30$

2 a) $\left[\frac{1}{2}\sin 2x\right]_0^{\pi/2} = 0$

b)



Shaded areas equal in size but + value above x axis cancelled out by − value below axis in integral

3. $p = \sqrt{q} \Rightarrow q = 25$

4. $2x^{-1/2} - 6\sin 2x$

5. a)

$$\begin{array}{c|cccc} & 2 & 1 & -13 & 6 \\ 2 & & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & \boxed{0} \end{array}$$

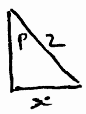
$f(2) = 0 \Rightarrow 2$ is root of equation

b) $(x-2)(2x^2+5x-3) = 0$

$(x-2)(2x-1)(x+3) = 0$

$\Rightarrow$ other roots $x = 1/2,\ x = -3$.

6. a)(i)



$\sin p = \frac{x}{2}$

$\Rightarrow x = 2\sin p$

OB = radius of upper ⊙ + x

$= 1 + 2\sin p$.

(ii) $OC = 1 + 2\cos p$

$d^2 = OC^2 + OB^2$

$= 1^2 + 4\sin p + 4\sin^2 p$

$+ 1 + 4\cos p + 4\cos^2 p$

$= 2 + 4(\sin^2 p + \cos^2 p) + 4\cos p$

$\sin^2 p + \cos^2 p = 1$ $\qquad + 4\sin p$

$= 6 + 4\cos p + 4\sin p$

b) (i) $4\cos p + 4\sin p = k\cos(p-\alpha)$

$= k\cos p\cos\alpha + k\sin p\sin\alpha$

$\Rightarrow k\cos\alpha = 4$

$k\sin\alpha = 4$ $\quad \Rightarrow k = 4\sqrt{2}$

b)(i) cont'd

$d = 6 + 4\sqrt{2}\cos(p - \pi/4)$

(ii) Max value $= 6 + 4\sqrt{2}$

when $p = \pi/4$

c) (i) $OB = 1 + 2\sin \pi/4$

$= 1 + 2 \cdot \frac{1}{\sqrt{2}}$

$= 1 + \sqrt{2}$.

$BD = \sqrt{\frac{1}{2}(1+\sqrt{2})^2}$

$= \frac{1+\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} + 1.$

(ii) $d = \sqrt{6+4\sqrt{2}} = 2BD$

$= \sqrt{2} + 2$

7. $x = \log_5 12$

$\Rightarrow 5^x = 12$

$\Rightarrow x\log_{10} 5 = \log_{10} 12$

$\Rightarrow x = \frac{\log_{10} 12}{\log_{10} 5} = 1.544$

8. $2x^3 - \frac{1}{2}x^2 + \sin x + C.$

9. $f(x) = \frac{1}{2}\sin 2x + c$

$f(\pi/12) = \frac{1}{2}\sin \pi/6 + c = 1$

$\Rightarrow c = 3/4$.

$f(x) = \frac{1}{2}\sin 2x + 3/4$.

10. $\frac{dy}{dx} = 6x^2 + 6x + 4$

$= 6(x^2 + x + 2/3)$

$= 6(x^2 + x + \frac{1}{4} - \frac{1}{4} + \frac{2}{3})$

$= 6(x + 1/2)^2 - \frac{6}{4} + 4$

$= 6(x + 1/2)^2 + 2\tfrac{1}{2} > 0 \ \forall x$

$\Rightarrow$ no T.P.s.

11. $\cos x = \pm 1/2$

$\Rightarrow x = \pi/3,\ 2\pi/3$

12. $f'(x) = x^2 - 4x - 5$

$= (x-5)(x+1)$

	$\to$	-1	$\to$	5	$\to$
$x-5$	$-$	$-$	$-$	0	$+$
$x+1$	$-$	0	$+$	$+$	$+$

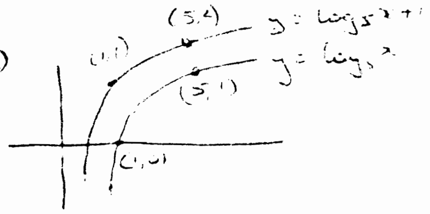
increasing for

$x < -1$ or $x > 5$

13. $\frac{d}{dx}(\sin^3 x) = 3\sin^2 x \cos x$

$\Rightarrow \int \sin^2 x \cos x \, dx = \frac{1}{3}\sin^3 x.$

14. a)



At intersection with x axis

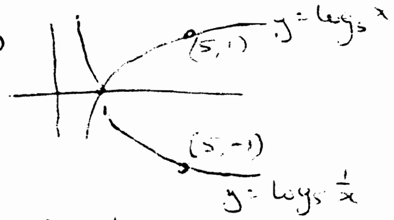
$\log_5 x + 1 = 0$

$\Rightarrow \log_5 x = -1$

$\Rightarrow x = 5^{-1} = \frac{1}{5}$

$\Rightarrow$ Point is $(1/5, 0)$

b)



$\log_5 \frac{1}{x} = \log_5 x^{-1}$

$= -\log_5 x.$

15. $f'(x) = -2\cos x \sin x - 2\sin x \cos x.$

$= -2\sin 2x$

16. a) $(\cos x + \sin x)^2 = \cos^2 x + 2\cos x \sin x + \sin^2 x$

$= \underbrace{\cos^2 x + \sin^2 x}_{} + 2\cos x \sin x$

$= 1 + \sin 2x.$

b) $\int (\cos x + \sin x)^2 dx = \int 1 + \sin 2x \, dx$

$= x - \frac{1}{2}\cos 2x + c$

17 a) $\sin 3x = \sin(2x + x)$

$= \sin 2x \cos x + \cos 2x \sin x$

$= 2\sin x \cos x \cos x + (1 - 2\sin^2 x)\sin x$

$= 2\sin x(1 - \sin^2 x) + (1 - 2\sin^2 x)\sin x$

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$= 3\sin x - 4\sin^3 x.$

b) $4\sin^3 x = 3\sin x - \sin 3x$

$\Rightarrow \int \sin^3 x \, dx = \frac{1}{4}\left(-3\cos x + \frac{1}{3}\cos 3x + c\right)$

18. $\frac{dy}{dx} = 10x$

Equation $y - b = m(x - a)$

$y - 7 = -10(x - -1)$

$10x + y + 3 = 0$

19 a) $g(x) = \cos x$ - even - symmetrical about line Oy.

$h(x) = \sin 2x$ - odd - $\frac{1}{2}$ turn symm. about O.

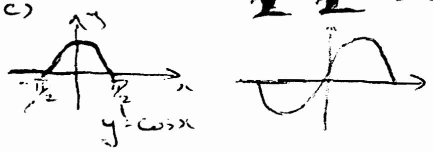
b) $\int_{-\pi/2}^{\pi/2} \cos x \, dx = [\sin x]_{-\pi/2}^{\pi/2}$

$= 1 - -1 = 2.$

$\int \sin 2x \, dx = \left[-\frac{1}{2}\cos 2x\right]_{-\pi/2}^{\pi/2}$

$= \frac{1}{2} - \frac{1}{2} = 0.$

c)



d) $v(-x) = -x\cos(-x)$

$= -x\cos x$ $\underline{\underline{\text{ODD}}}$

$\Rightarrow \int_{-\pi/2}^{\pi/2} x\cos x \, dx = 0$

20. $\underline{\text{Area 1}}$ $\int_{\pi/6}^{\pi/4} \cos 2x \, dx = \left[\frac{1}{2}\sin 2x\right]_{\pi/6}^{\pi/4}$

$= \frac{1}{2} - \frac{\sqrt{3}}{4} = \frac{2 - \sqrt{3}}{4}$

$\underline{\text{Area 2}}$ $\int_{\pi/4}^{\pi/2} \cos 2x \, dx = \left[\frac{1}{2}\sin 2x\right]_{\pi/4}^{\pi/2}$

$= 0 - \frac{1}{2} \Rightarrow$ Area $= \frac{1}{2}$

$\Rightarrow$ Total area

21. $f(x) = 6x^3 - 3x^2$

$f'(x) = 18x^2 - 6x.$

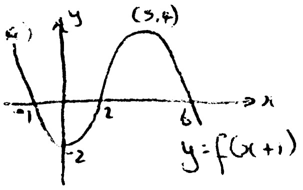
22. $PR = \sqrt{11}$

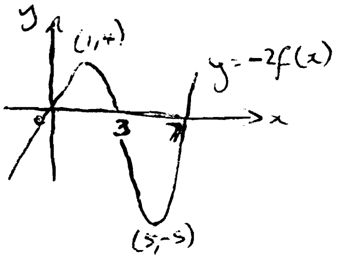
$\cos 2x = \cos^2 x - \sin^2 x.$

$= \frac{4}{11} - \frac{7}{11}$

$= -\frac{3}{11}$

23 a)





24. $y = \int x^3 + x^{-2} - \frac{1}{4}\, dx$

$= \frac{1}{4}x^4 - x^{-1} - \frac{1}{4}x + c$

at $(1,2)$

$2 = \frac{1}{4} - 1 - \frac{1}{4} + c \Rightarrow c = 3$

$\Rightarrow y = \frac{1}{4}x^4 - x^{-1} - \frac{1}{4}x + 3.$

25. $\int x^{1/2} - 5x^{-3/2}\, dx$

$= \frac{2}{3}x^{3/2} + 10x^{-1/2} + C$

26. $f'(x) = -3x^{-4} - \frac{1}{3}\sin 3x.$

27. a) Area $= 3 \times \left(\frac{8+14}{2}\right) = 33$ units^2.

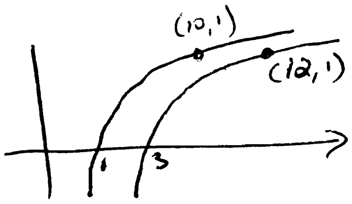
b) $\int_2^5 2x + 4\, dx$

c) $= \left[x^2 + 4x\right]_2^5$

$= (25 + 20) - (4 + 8)$

$= 33.$

28



29. $f'(x) = 2(\sin x + 1).\cos x$

$f'(\pi/6) = 2\left(\frac{1}{2} + 1\right).\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

30. a) $2(x^2 - 2x) + 5 = 2(x^2 - 2x + 1 - 1) + 5$

$= 2(x-1)^2 - 2 + 5$

$= 2(x-1)^2 + 3$

b) SP $(1, 3)$

31. $\sin 3x = 2/3 \Rightarrow 3x = 41.8$ or 138.2

$\Rightarrow x = 13.9$ or 46.1

32. $y = (1 + \cos x)^{1/2}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(1 + \cos x)^{-1/2} \times -\sin x.$

$= \frac{-\sin x}{2(1 + \cos x)^{1/2}}$

33. $2x + 3y - 4 = 0$

$x - 3y - 10 = 0$

Add $3x - 14 = 0.$

$x = 14/3 \Rightarrow y = \frac{-16}{9}$

Substitute in 2nd eqn

$3x - y - 17 = 14 + \frac{16}{9} - 17 \neq 0$

$\Rightarrow$ Not concurrent.

34. $x(x^3 - 1)$

$= \underline{x(x-1)(x^2+x+1)}$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 0 & -1 \\ & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & \boxed{0} \end{array}$$

35. $\frac{dy}{dx} = 12x^2 \Rightarrow m = 12.$

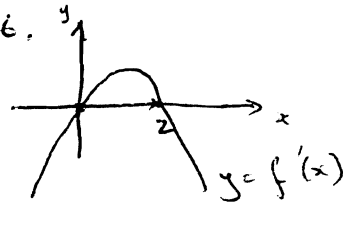
$x = -1 \Rightarrow y = -6$

$y - b = m(x - a)$

$y - -6 = 12(x - -1)$

$\underline{y = 12x + 6.}$

36.



37. $f'(x) = -2\cos x \sin x + \frac{4}{3}x^{-3}$

38. $y = \int 1 - 2x\, dx = x - x^2 + c$

$\Rightarrow y = 5 + x - x^2$

39. $\cos D = \frac{2}{\sqrt5}$

$\sin D = \frac{1}{\sqrt5}$

$\sin 2D = 2\sin D\cos D$

$= \frac{4}{5}$

$\cos 2D = 2\cos^2 D - 1$

$= 3/5$

40 a) $2x^2 = 4 - 2x^2$

$\Rightarrow 4x^2 = 4$

$\Rightarrow x = \pm 1$

$\Rightarrow$ Intersections are $(1,2)$ and $(-1,2)$

b) $\int_{-1}^{1} 4 - 2x^2 - 2x^2 \, dx$

$= \int_{-1}^{1} 4 - 4x^2 \, dx$

$= \left[4x - \frac{4}{3}x^3\right]_{-1}^{1}$

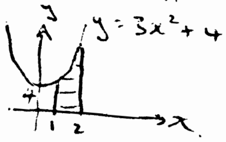
$= \left(4 - \frac{4}{3}\right) - \left(-4 + \frac{4}{3}\right)$

$= 5\frac{1}{3}$

$\int_1^2 3x^2 + 4 \, dx = \left[x^3 + 4x\right]_1^2$

$= (8+8) - (1+4)$

$= 11$



$(1+3x)^{\frac12} dx$

$= \frac{2}{3}(1+3x)^{3/2} \cdot \frac{1}{3} + c$

$= \frac{2}{9}(1+3x)^{3/2} + c$

$(1+3x)^{\frac12} dx = \left[\frac{2}{9}(1+3x)^{3/2}\right]_0^1$

$= \frac{16}{9} - \frac{2}{9}$

$= \frac{14}{}$

43. a) $k = 3$

b) At P

$y = 3e^{0.5} \approx 4.95$

$P(1, 4.95)$.

44. a) $M(6, 15, 12)$

$g = \frac{1}{3}p + \frac{2}{3}m$

$= \begin{pmatrix}9\\1\\0\end{pmatrix} + \begin{pmatrix}4\\12\\8\end{pmatrix}$

$G(13, 13, 8)$

b) $a \cdot g = 117 + 117 + 192$

$= 426$

$|a| = \sqrt{81+81+576} = \sqrt{738}$

$|g| = \sqrt{169+169+64} = \sqrt{402}$

$\cos G\hat{O}A = \frac{a \cdot g}{|a||g|} = \frac{426}{\sqrt{738}\sqrt{402}}$

$G\hat{O}A = 38.5°$.

45 a) $k\sin(x-\alpha) = k\sin x\cos\alpha - k\cos x\sin\alpha$

$\sin x \quad - 3\cos x$

$\Rightarrow k\cos\alpha = 1$

$k\sin\alpha = 3$

$\underline{k = \sqrt{10}}$ $\quad \tan\alpha = \frac{3}{1} \Rightarrow \underline{\alpha = 71.6}$

b) Max. value $= 5 + \sqrt{10}$

when $x - 71.6 = 90 \Rightarrow x = 161.6$

46. $k\sin x = 5$

$k\cos x = 2$ $\quad \Rightarrow \tan x = 5/2$

$\Rightarrow x = \underline{68.2}$

$k = 5/\sin 68.2 = \underline{5.39}$

47 a) $\underline{a = 5}$. $\quad$ at $x = 3$, $\quad 20 = 5e^{k \times 3}$

$\Rightarrow 4 = e^{3k}$

$\Rightarrow 3k = \log_e 4$

$\Rightarrow k = \frac{\log_e 4}{3} = \underline{0.462}$.

b) $y = 5e^{0.462(x-3)}$

48. a) $\sqrt{(7-5)^2 + (-4-5)^2 + (7.5 - -0.5)^2} = \sqrt{149}$ units

$= \underline{2\sqrt{149}\text{ km}}$

b) $\sqrt{(7 - -2)^2 + (-4-4)^2 + (7.5-8.5)^2}$

$= \sqrt{146}$ units in 3 mins

$= 2\sqrt{146}$ km in 3 mins.

48 d) $\vec{CT} = \begin{pmatrix} -14 \\ -4 \\ -1 \end{pmatrix}$

$\vec{CR} = \begin{pmatrix} -7 \\ -8 \\ 6.5 \end{pmatrix}$

$\vec{CT} \cdot \vec{CR} = 98 + 32 - 6.5$

$= 123.5$

$|\vec{CT}| = \sqrt{(-14)^2 + (-4)^2 + (-1)^2}$

$= \sqrt{213}$

$|\vec{CR}| = \sqrt{(-7)^2 + (-8)^2 + (6.5)^2}$

$= \sqrt{155.25}$

$\cos T\hat{C}R = \dfrac{\vec{CT} \cdot \vec{CR}}{|\vec{CT}||\vec{CR}|}$

$= \dfrac{123.5}{\sqrt{213}\sqrt{155.25}}$

$T\hat{C}R = \cos^{-1} 0.679 = \underline{47.2^\circ}$

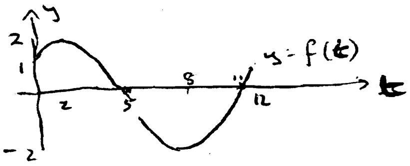
49. a) $k\cos(30t - \alpha) = k\cos 30t \cos\alpha + k\sin 30t \sin\alpha$

$k\cos\alpha = 1$
$k\sin\alpha = \sqrt{3}$ $\Rightarrow k^2 = 1^2 + \sqrt{3}^2$
$\Rightarrow \underline{k = 2}$

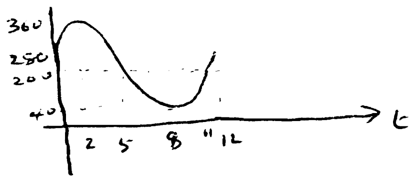
$\dfrac{k\sin\alpha}{k\cos\alpha} = \dfrac{\sqrt{3}}{1} \Rightarrow \alpha = \tan^{-1}\sqrt{3}$
$= \underline{60^\circ}$

$f(t) = 2\cos(30t - 60)^\circ$

b)(i)



(ii)



c) 8 am.

d) $150 = 200 + 80 \times 2\cos(30t - 60)$

$\Rightarrow \cos(30t - 60) = \dfrac{-50}{160}$

$\Rightarrow 30t - 60 = 180 \pm 33.6$

$\Rightarrow t = 6.88$ or 9.12

$\Rightarrow$ Do not enter between approx 6.50 am and 9.10 am

50. a) $A(1, 0, 0)$
$B(3, 2, 0)$
$C(3, 0, -2)$

b) $|\vec{AB}| = \sqrt{2^2 + 2^2 + 0^2} = \sqrt{8}$

$\Rightarrow$ Area of $\Delta = \frac{1}{2} bc \sin A$
$= \frac{1}{2}\sqrt{8}\sqrt{8}\sin 60$
$= 2\sqrt{3}$ units^2

c). Surf. Area of Cube $= 6 \times 3 \times 3$
$= 54$ units^2

Area of top of new shape $= (3 \times 3) - \frac{1}{2}(2 \times 2)$
$= 7$ units^2

$\Rightarrow$ Surf area of new shape $=$
$3 \times 9 + 3 \times 7 + 2\sqrt{3}$
$= 48 + 2\sqrt{3}$.

$\Rightarrow$ New area $= \dfrac{48 + 2\sqrt{3}}{54} \times$ old area

$= 0.953 \times$ old area

$\Rightarrow$ 4.7 % decrease in area

51. $3^x = 42$

$\Rightarrow x \log 3 = \log 42$

$\Rightarrow x = \dfrac{\log 42}{\log 3} = 3.402$

$\Rightarrow P$ is $(3.402, 42)$

52. $\dfrac{dy}{dx} = -\dfrac{5}{2}\cos\frac{1}{2}x$.

At $\dfrac{7\pi}{3}$, $m_{TAN} = -\dfrac{5}{2}\cos\dfrac{7\pi}{6}$

$= 2.165$

$\Rightarrow$ Angle between floor & horizontal $= \tan^{-1} 2.165$

$= 65.2^\circ$.

53. a) $9 = 10e^{-k \times 3}$

$\Rightarrow e^{-3k} = 0.9$

$\Rightarrow -3k = \log_e 0.9$

$\Rightarrow k = \dfrac{\log_e 0.9}{-3} = 0.035$

b) For ½ life $e^{-0.035t} = 0.5$

$\Rightarrow t = \dfrac{\log_e 0.5}{-0.035} = 19.7$ min

a) $2\cos(x+30) - \sin x$

$= 2(\cos x\cos 30 - \sin x \sin 30) - \sin x$

$= 2\left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right) - \sin x$

$= \sqrt{3}\cos x - \sin x - \sin x$

$= \sqrt{3}\cos x - 2\sin x$

b) $R\cos(x+\alpha)$

$= R\cos x\cos\alpha - R\sin x\sin\alpha$

$\Rightarrow R\cos\alpha = \sqrt{3}$

$R\sin\alpha = 2$

$\underline{R = \sqrt{7}}$ $\tan\alpha = \frac{2}{\sqrt{3}}$

$\Rightarrow \underline{\alpha = 49.1^\circ}$

$\sqrt{3}\cos x - 2\sin x$

$= \sqrt{7}\cos(x + 49.1)$

c) $2\cos(x+30) - \sin x = 1$

$\Rightarrow \sqrt{7}\cos(x+49.1) = 1$

$\cos(x+49.1) = \frac{1}{\sqrt{7}}$

$\Rightarrow x + 49.1 = 67.8$ or 292.2

$\Rightarrow \underline{x = 18.7}$ or $\underline{243.1}$

55 a) $\vec{AB} = \underline{b} - \underline{a} = \begin{pmatrix} 3-2 \\ 6-\cancel{1} \\ 5-3 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$

$\vec{AC} = \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix}$

b) $\vec{AB}\cdot\vec{AC} = 4 + 49 - 10 = 43$

$|\vec{AB}| = \sqrt{54}$

$|\vec{AC}| = \sqrt{90}$

$\Rightarrow \cos\hat{A} = \frac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}||\vec{AC}|} = \frac{43}{\sqrt{54}\sqrt{90}}$

$\hat{A} = 51.9^\circ$

c) Area $= \frac{1}{2}bc\sin A$

$= \frac{1}{2}\sqrt{54}\sqrt{90}\sin 51.9$

$= 27.4$ units^2

56 a)

$75 = 100e^{-k \times 15}$

$e^{-15k} = 0.75$

$-15k = \log_e 0.75$

$k = \frac{\log_e 0.75}{-15} = -0.019$

b) $\frac{1}{4}$ of temperature lost in 15 mins

$\Rightarrow 18.75^\circ$ fall in next 15 min

57. $R\sin(x-\alpha) = R\sin x\cos\alpha - R\cos x\sin\alpha$

$\Rightarrow R\cos\alpha = 2$

$R\sin\alpha = 5$

$\underline{R = \sqrt{29}}$ $\tan\alpha = \frac{5}{2}$

$\alpha = \underline{68.2^\circ}$

58 a) $\int_0^2 2x - x^2\,dx = \left[x^2 - \frac{1}{3}x^3\right]_0^2$

$= \left(4 - \frac{8}{3}\right) - 0$

$= 1\frac{1}{3}$ units^2.

b) Area $= \frac{1}{2}p \cdot \frac{1}{2}p$

$\Rightarrow \frac{1}{4}p^2 = \frac{4}{3}$

$\Rightarrow p^2 = \frac{1}{3}$

$\Rightarrow p = \frac{1}{\sqrt{3}}$

c) $\int_{\pi/4}^{q} \sin x - \cos x\,dx$

$= \left[-\cos x - \sin x\right]_{\pi/4}^{q}$

$\Rightarrow \left(-\cos q - \sin q\right) - \left(-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right)$

$= 1\frac{1}{3}$

$\Rightarrow -(\cos q + \sin q) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) = 1\frac{1}{3}$

$\Rightarrow \cos q + \sin q = \frac{2}{\sqrt{2}} - 1\frac{1}{3} = 0.081$

$\cos q + \sin q = R\cos(q - \alpha)$

$= R\cos q\cos\alpha + R\sin q\sin\alpha$

$\Rightarrow R\cos\alpha = 1$ $\Rightarrow R = \sqrt{2}$

$R\sin\alpha = 1$

59. a) $k\cos x\cos\alpha + k\sin x\sin\alpha$

$k\cos\alpha = 2$

$k\sin\alpha = 3$

$k = \sqrt{13}$ $\quad \tan\alpha = 3/2$

$\Rightarrow \alpha = 56.3°$

$\Rightarrow f(x) = \sqrt{13}\cos(x - 56.3)$

b) $\cos(x - 56.3) = \dfrac{0.5}{\sqrt{13}}$

$x - 56.3 = 82.0$ or $(360 - 82.0)$

$x = 138.3$ or 334.3

60 a) Using end points $m = \dfrac{10-7}{3.1-2.1} = 3.$

Eqn $Y = 3X + 0.7$

b) $\log_e y = 3\log_e x + 0.7$

$\log_e y = \log_e x^3 + \log_e e^{0.7}$

$\log_e y = \log_e x^3 + \log_e 2.01$

$y = 2.01x^3.$

61. a) $\underset{\sim}{r} - \underset{\sim}{p} = \frac{4}{3}(\underset{\sim}{q} - \underset{\sim}{p})$

$\underset{\sim}{r} = \frac{4}{3}\underset{\sim}{q} - \frac{4}{3}\underset{\sim}{p} + \underset{\sim}{p}$

$= \begin{pmatrix} \frac{20}{3} \\ 0 \\ \frac{20}{3} \end{pmatrix} - \begin{pmatrix} -\frac{1}{3} \\ 3/3 \\ 2/3 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ 1 \\ 6 \end{pmatrix}$

R is $(7, 1, 6)$

b) $\overrightarrow{SP} = \underset{\sim}{p} - \underset{\sim}{s} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$

$\overrightarrow{SR} = \begin{pmatrix} 9 \\ -1 \\ 1 \end{pmatrix}$

$\overrightarrow{SP}.\overrightarrow{SR} = 9 + -1 + -3 = 5$

$|\overrightarrow{SP}| = \sqrt{11}$ $\quad |\overrightarrow{SR}| = \sqrt{83}$

$\cos S = \dfrac{5}{\sqrt{11}\sqrt{83}}$

62. a) $k\cos(20t - \alpha)$

$= k\cos 20t\cos\alpha + k\sin 20t\sin\alpha$

$\Rightarrow k\cos\alpha = 1$

$k\sin\alpha = \sqrt{3}$

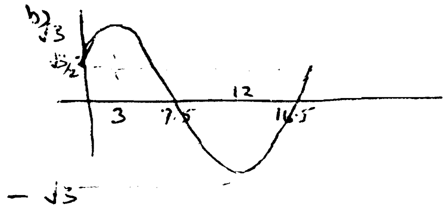
$k = 2$

$\tan\alpha = \sqrt{3}$

$\Rightarrow \alpha = 60°$

$d = \sqrt{3}\cos(20t - 60)°$

b)



c) $1.5 = \sqrt{3}\cos(20t - 60)$

$\cos(20t - 60) = \dfrac{1.5}{\sqrt{3}}$

$20t - 60 = 30, 330, -30$ etc

$t = 1.5, 4.5.$

63. $y = 5\log_{10}(2x + 10)$

$y = 0 \Rightarrow 5\log_{10}(2x+10) = 0$

$\Rightarrow 2x + 10 = 1$

$\Rightarrow x = -4\frac{1}{2}$

$y = 8 \Rightarrow 5\log_{10}(2x+10) = 8$

$\Rightarrow 2x + 10 = 10^{8/5}$

$\Rightarrow 2x = 29.8$

$x = 14.9$

$A(-4.5, 0)$ $\quad B(14.9, 8)$

64. $\cos R\hat{O}C = \frac{5}{13} \Rightarrow R\hat{O}C = 67.4°$

$\cos B\hat{O}A = 4/5 \Rightarrow B\hat{O}A = 36.9$

$R\hat{O}A = 104.3°$

$\Rightarrow m_{RO} = \tan 104.3 = -3.92.$

65 a) $k\cos(x + \alpha)$

$= k\cos x\cos\alpha - k\sin x\sin\alpha$

$\Rightarrow k\cos\alpha = 2$ $\quad k\sin\alpha = 3$

$k = \sqrt{13}$ $\quad \tan\alpha = \frac{3}{2} \Rightarrow \alpha = 56.3$

$\Rightarrow f(x) = \sqrt{13}\cos(x + 56.3)$

b) max $\sqrt{13}$ when $x = 303.7°$

min $-\sqrt{13}$ when $x = 123.7°$

c) min $(f(x))^2 = 0$

66. a) $10 = 50e^{-k\times 24}$

$e^{-24k} = 0.2$

$-24k = \log_e 0.2$

$k = \dfrac{\log_e 0.2}{-24} = 0.067$

b) $P_t = 50e^{-0.067\times 4}$

$= 50 \times 0.765$

$= 38.2$

> 30

$\Rightarrow$ Tyre can be driven on

67. $Q(2,2,9)$

$R(21,3,12)$

$\vec{PQ} = q - p = \begin{pmatrix} -10 \\ 2 \\ 9 \end{pmatrix}$

$\vec{PR} = r - p = \begin{pmatrix} 9 \\ 3 \\ 12 \end{pmatrix}$

$\vec{PQ}.\vec{PR} = -90 + 6 + 108$

$= 24$

$|\vec{PQ}| = \sqrt{185}$

$|\vec{PR}| = \sqrt{252}$

$\cos \vec{QPR} = \dfrac{24}{\sqrt{185}\sqrt{252}}$

$Q\hat{P}R = 83.6°$

68a) $k\sin(x - \alpha) = k\sin x\cos\alpha - k\cos x\sin\alpha$

$k\cos\alpha = 3$ $\quad k = \sqrt{10}$

$k\sin\alpha = 1$ $\quad \tan\alpha = 1/3$

$\Rightarrow \alpha = 18.4°$

$3\sin x - \cos x = \sqrt{10}\sin(x - 18.4)°$

b) $\sqrt{10}\sin(x - 18.4) = \sqrt{5}$

$\sin(x - 18.4) = \dfrac{\sqrt{5}}{\sqrt{10}} = \dfrac{1}{\sqrt{2}}$

$x - 18.4 = 45$ or 135

$x = 63.4$ or 153.4

c) $3\sin x - \cos x \leq \sqrt{5}$ for

$x < 63.4$ or $x > 153.4$

69a) $\log_e y = \log_e a e^{bx}$

$= \log_e e^{bx} + \log_e a$

$= bx + \log_e a$.

$\Rightarrow$ linear relationship between $\log_e y$ and x.

b,

x	3.1	3.5	4.1	5.2
$\log_e y$	9.993	11.193	12.993	16.293

$m = \dfrac{12.993 - 9.993}{4.1 - 3.1} = 3 = b$

$\Rightarrow \log_e y = 3x + \log_e a$

$\Rightarrow \log_e a \quad 0.693$

$\Rightarrow a = e^{0.693} = 2$.

70a) $B(6,4,2)$ $\quad C\ (4,3,4)$

$D\ (6,2,2)$

b) mid pt of AD $= \left(\dfrac{2+6}{2}, \dfrac{4+2}{2}, \dfrac{6+2}{2}\right)$

$= (4,3,4) = C$

c) $|\vec{OA}| = \sqrt{56}$ $\quad |\vec{OB}| = \sqrt{56}$

$\vec{OA}.\vec{OB} = 12 + 16 + 12 = 40$

$\Rightarrow \cos A\hat{O}B = \dfrac{40}{\sqrt{56}\sqrt{56}}$

$A\hat{O}B = 44.4°$.

d) $OA = OB \Rightarrow O\hat{A}B = 67.8°$

71. a) $e^{5700k} = 0.5$

$5700k = \log_e 0.5$

$k = \dfrac{\log_e 0.5}{5700} = -1.22\times10^{-4}$

b) $e^{-1.22\times10^{-4}\times1000}$

$= 0.885$

$\Rightarrow 88.5\%$ remains after 1000 years.

72. a) (i) $b = 2$, $a = 3$

(ii) $d = 3$, $c = 3$

b) $h(x) = 2\sin 3x + 3\cos 3x.$

$q\sin(3x + r)$

$= q\sin 3x\cos r + q\cos 3x\sin r$

$q\cos r = 2$

$q\sin r = 3$

$\underline{p = 3}$

$\underline{q = \sqrt{13}}$

$\tan r = 3/2$

$\Rightarrow r = 56.3°$

73. a) $m = -0.8$ $c = 4$

$\Rightarrow \log_e I = -0.8\log_e t + 4$

b) $\Rightarrow \log_e I = \log_e t^{-0.8} + \log_e e^4$

$\Rightarrow I = 54.6\,t^{-0.8}$

$\underline{k = 54.6 \quad r = -0.8}$

74. a) $y = 2\sin 4x°.$

b) $-1.5 = 2\sin 4x$

$\sin 4x = -\frac{1.5}{2}$

$4x = 180 + 48.6$ or $360 - 48.6$

$x = 57.1$ or 77.9

75. a) side of square $= 4\sin\theta/2.$

$\Rightarrow$ area of square $= 16\sin^2\theta/2$

$=$

$= 8\left(1 - \cos 2\times\frac{\theta}{2}\right)$

$= 8(1 - \cos\theta)$

area $\Rightarrow \Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2}\times 4\times 4\sin\theta$

$= 8\sin\theta.$

$\Rightarrow$ Total area $= 8 - 8\cos\theta + 8 - 8\cos\theta + 8\sin\theta$

$= 8(2 + \sin\theta - 2\cos\theta)$

b) $k\sin(\theta - \alpha) = k\sin\theta\cos\alpha - k\cos\theta\sin\alpha$

$\Rightarrow k\cos\alpha = 8$

$k\sin\alpha = 16$

$k = 8\sqrt{5}$

$\tan\alpha = \frac{16}{8}$

$\alpha = 63.4$

$\Rightarrow 8\sin\theta - 16\cos\theta = 8\sqrt{5}\sin(\theta - 63.4)$

75c) $30 = 16 + 8\sqrt{5}\sin(\theta - 63.4)$

$\Rightarrow \sin(\theta - 63.4) = \frac{14}{8\sqrt{5}}$

$\Rightarrow \theta - 63.4 = 51.5$, ~~128.5~~

$\Rightarrow \theta = 114.9°$, ~~reflex~~

76a) $m = 500e^{-0.02\times 10}$

$= 409.4\,g.$

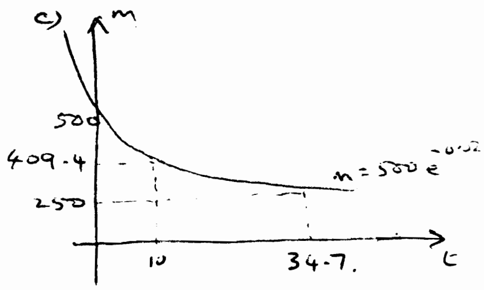
b) $e^{-0.02t} = 0.5$

$-0.02t = \log_e 0.5$

$t = \frac{\log_e 0.5}{-0.02}$

$= 34.7$ years.

c)



77. a) $\vec{AB} = \underline{b} - \underline{a}$

$= \begin{pmatrix}-1\\3\\2\end{pmatrix} - \begin{pmatrix}-3\\2\\4\end{pmatrix}$

$= \begin{pmatrix}2\\1\\-2\end{pmatrix}$

b) $|\vec{AB}| = \sqrt{2^2 + 1^2 + (-2)^2}$

$= \sqrt{9} = \underline{3}$

78. $\underline{a}.\underline{b} + \underline{a}.\underline{c}$

$= \begin{pmatrix}2\\0\\-1\end{pmatrix}.\begin{pmatrix}1\\2\\1\end{pmatrix} + \begin{pmatrix}2\\0\\-1\end{pmatrix}\begin{pmatrix}0\\-1\\1\end{pmatrix}$

$= 2 + 0 + -1 + 0 + 0 + -1$

$= 0$

b) $\underline{a}.(\underline{b} + \underline{c}) = 0$

$\Rightarrow \underline{b} + \underline{c} \perp \underline{a}$

79. $a = -2$

$b = 5$

80a(i) $\underline{a}.\underline{a} = 9$

(ii) $\underline{b}.\underline{b} = 8$

(iii) $\underline{a}.\underline{b} = 6\sqrt{2}\cos 45$

$= 6$

b) $\underline{p}.\underline{p} = (2\underline{a}+3\underline{b})(2\underline{a}+3\underline{b})$

$= 4\underline{a}.\underline{a} + 12\underline{a}.\underline{b} + 9\underline{b}.\underline{b}$

$= 36 + 72 + 72$

$= 180$

$\Rightarrow |\underline{p}| = \sqrt{180} = 6\sqrt{5}$

81. $\underline{a}.(\underline{a}+\underline{b}) = \underline{a}.\underline{a} + \underline{a}.\underline{b}$

$= 4 + 6\cos 60$

$= 7.$

82. $\underline{a}.(\underline{b}+\underline{c}) = \underline{a}.\underline{b} + \underline{a}.\underline{c}$

$= 2\times 2\cos 60$

$+ 2\times 2\cos 120$

$= 2 + -2$

$= 0$

$\Rightarrow \underline{a} \perp (\underline{b}+\underline{c})$

83. Body has centre $(5,6)$

radius $\sqrt{5^2+6^2-45} = 4$

$\Rightarrow$ diameter of body $= 8$

$\Rightarrow$ diameter of head $= 6$

$\Rightarrow$ centre of head $(5,13)$

Eqn of head

$(x-5)^2+(y-13)^2 = 3^2$

$x^2+y^2-10x-26y+185=0$

84a) $\vec{AB} = \begin{pmatrix}9\\2\\-4\end{pmatrix} - \begin{pmatrix}3\\-1\\2\end{pmatrix} = \begin{pmatrix}6\\3\\-6\end{pmatrix}$

$\vec{AC} = \begin{pmatrix}2\\1\\-2\end{pmatrix}$

b) C $(5,0,0)$

D $(7,1,-2)$

85. $m_{AB} = \dfrac{-1-3}{4--2} = -\dfrac{2}{3}.$

$\Rightarrow m_{DIAM} = \frac{3}{2}.$

mid pt of AB $(1,1)$

Eqn of diam. $y-b = m(x-a)$

$y-1 = \frac{3}{2}(x-1)$

$2y = 3x-1$

86. $b=2$

$a=3.$

87. $1\times -4 + 2\times 3 + (-1)(k-1)$

$= -4+6-k+1$

$\Rightarrow 3-k = 0 \Rightarrow \underline{k=3}$

88a) $\vec{PQ} = \underline{q}-\underline{p} = \begin{pmatrix}7\\-1\\5\end{pmatrix} - \begin{pmatrix}-1\\3\\4\end{pmatrix}$

$= \begin{pmatrix}8\\-4\\1\end{pmatrix}$

b) $|\vec{PQ}| = \sqrt{64+16+1} = 9$

89.

$$\begin{array}{r|rrrr} & 1 & -1 & p & 15 \\ -3 & & -3 & 12 & -3p-36 \\ \hline & 1 & -4 & p+12 & -3p-21 = 0 \end{array}$$

$p = -7.$

90. $\vec{AB} = \begin{pmatrix}1\\-4\\2\end{pmatrix}$

$\vec{BC} = \begin{pmatrix}2\\-8\\4\end{pmatrix} = 2\vec{AB}$

$\Rightarrow$ BC & AB are parallel

But B common

$\Rightarrow$ A, B, C collinear

91. Disc $= b^2-4ac$

$= [-(3k-2)]^2 - 4(k-2).2k$

$= 9k^2-12k+4-8k^2+16k$

$= k^2-4k+4$

$= (k-2)^2 \geq 0$ for all k

$\Rightarrow$ Roots of eqn real.

92. Centre of A $(-12, -15)$

Radius of A: 5

Centre of C $(24, 12)$

Radius of C $= 10$

$AC = \sqrt{36^2 + 27^2} = 45$

$\Rightarrow$ Diam of B is 30

$\Rightarrow BC = 25$

$AB = 20$

$\Rightarrow$ B divides AC in ratio 4:5

$\underline{b} = \frac{5}{9}\begin{pmatrix}-12\\-15\end{pmatrix} + \frac{4}{9}\begin{pmatrix}24\\12\end{pmatrix}$

B is $(4, -3)$

Eqn of central $\odot$ $(x-4)^2 + (y-3)^2 = 225$

93. radius $= 5$

Eqn $(x+3)^2 + (y-4)^2 = 25$

94. $\overrightarrow{LM} = \begin{pmatrix}7\\-2\\-1\end{pmatrix} - \begin{pmatrix}-5\\6\\-5\end{pmatrix} = \begin{pmatrix}12\\-8\\4\end{pmatrix}$

$\overrightarrow{MN} = \begin{pmatrix}10\\-4\\0\end{pmatrix} - \begin{pmatrix}7\\-2\\-1\end{pmatrix} = \begin{pmatrix}3\\-2\\1\end{pmatrix}$

$\overrightarrow{MN} = \frac{1}{4}\overrightarrow{LM}$

$\Rightarrow$ ~~MN~~ & LM parallel

but M common

$\Rightarrow$ L, M, N collinear

M divides LN in ratio 4:1

95. $\underline{p}\cdot(\underline{q}+\underline{r}) = \underline{p}\cdot\underline{q} + \underline{p}\cdot\underline{r}$

$= 3\times3\times\cos 60 + 3\times3\times\cos 60$

$= 9.$

96 a) $m_{AB} = \frac{5--1}{5--3} = \frac{3}{4}$

Eqn $y - b = m(x-a)$

$y - 5 = \frac{3}{4}(x-5)$

$4y = 3x + 5.$

b) mid-pt $(1, 2)$

$m_{BISECTOR} = -4/3$

Eqn $y - b = m(x-a)$

$y - 2 = -4/3(x-1)$

97. $\sqrt{2^2 + (-3)^2 + \sqrt{3}^2} = 4.$

98. $\underline{b}\cdot(\underline{a}+\underline{b}+\underline{c})$

$= \underline{b}\cdot 2\underline{b}$

$= 8.$

99 (i) $b^2 - 4ac = 0$

(ii) $x^2 + 7x = x - 9$

$\Rightarrow x^2 + 6x + 9 = 0$

$b^2 - 4ac = 36 - 4\times1\times9 = 0$

$\Rightarrow$ roots equal

100. a) $f(2x) = 1 + 2x + \frac{4x^2}{2} + \frac{8x^3}{6} + \frac{16x^4}{24} + \frac{32x^5}{120} + \dots$

$= 1 + 2x + 2x^2 + \frac{4x^3}{3} + \frac{2x^4}{3} + \frac{4x^5}{15} + \dots$

b) $f'(x) = 0 + 2 + 4x + 4x^2 + \frac{5x^3}{3} + \frac{4}{3}x^4 + \dots$

$= 2 + 4x + 4x^2 + \frac{8x^3}{3} + \frac{4x^4}{3} + \dots$

$= 2f(2x)$

101 a)(i) $\overrightarrow{VF} = \underline{f} - \underline{v} = \begin{pmatrix}2\\2\\7\end{pmatrix} - \begin{pmatrix}1\\1\\3\end{pmatrix}$

$= \begin{pmatrix}1\\1\\-10\end{pmatrix}$

$\overrightarrow{VE} = \underline{e} - \underline{v} = \begin{pmatrix}2\\0\\-7\end{pmatrix} - \begin{pmatrix}1\\1\\3\end{pmatrix}$

$= \begin{pmatrix}1\\-1\\-10\end{pmatrix}$

(ii) $\overrightarrow{VF}\cdot\overrightarrow{VE} = 1\times1 + 1\times-1 + -10\times-10$

$= 100$

$|\overrightarrow{VF}| = \sqrt{102}$ $\quad |\overrightarrow{VE}| = \sqrt{102}$

$\cos \hat{EVF} = \frac{100}{\sqrt{102}\sqrt{102}} \Rightarrow EVF = 11.4°$

b) Area of $\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2}\sqrt{102}\sqrt{102}\sin 11.4$

$= 10.05\ cm^2$

102. $L_u = \frac{p}{1-0.2} = \frac{p}{0.8}$

$L_V = \frac{q}{1-0.6} = \frac{q}{0.4}$

$\Rightarrow p = 2q$.

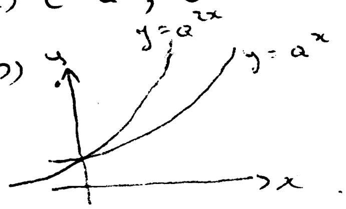
103. $\vec{CV} = \vec{CB} + \vec{BA} + \vec{AV}$

$= \begin{pmatrix} 2 \\ -10 \\ 2 \end{pmatrix} + \begin{pmatrix} -8 \\ -2 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 7 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} -5 \\ -5 \\ 7 \end{pmatrix}$

104. a) $t = a$, $u = 0$

b)



c) $(1, a^2)$

105. a) $M(6,1,1)$

b) $\underset{\sim}{t} = \frac{1}{3}\underset{\sim}{c} + \frac{2}{3}\underset{\sim}{m}$

$= \begin{pmatrix} 0 \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + \begin{pmatrix} 4 \\ 2/3 \\ 2/3 \end{pmatrix}$

$T(4,2,2)$

c) $\vec{BT} = \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix}$

$\vec{TD} = \begin{pmatrix} -8 \\ -2 \\ 6 \end{pmatrix} = 2\vec{BT}$

$\Rightarrow$ TD & BT parallel

but T common

$\Rightarrow$ B, T & D parallel

T divides BD in ratio 1:2

106. $\underset{\sim}{c} = \underset{\sim}{b} + \frac{1}{2}\vec{AB}$

$= \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 4 \\ -2 \\ -6 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 0 \\ -5 \end{pmatrix}$

107. $\underset{\sim}{a} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ $\underset{\sim}{b} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\underset{\sim}{a} \times \underset{\sim}{b} = \begin{pmatrix} 2\times -1 - -2\times 2 \\ -2\times -1 - 1\times -1 \\ 1\times 2 - 2\times -1 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$

$\underset{\sim}{a}\cdot\underset{\sim}{c} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}\cdot\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = 2+6+-8 = 0$

$\underset{\sim}{b}\cdot\underset{\sim}{c} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = -2+6+-4 = 0$

$\Rightarrow \underset{\sim}{c} \perp$ both $\underset{\sim}{a}$ & $\underset{\sim}{b}$.

108 a) $x^2 + y^2 = 20 \Rightarrow y = \sqrt{20-x^2}$

horizontal $= 2x$

$\Rightarrow$ length of $L = 2x + \sqrt{20-x^2}$

b) $\frac{dL}{dx} = 2 + \frac{1}{2}(20-x^2)^{-1/2} \times -2x$.

$= 2 - \frac{x}{\sqrt{20-x^2}} = 0$ at TP

$\Rightarrow 2 = \frac{x}{\sqrt{20-x^2}}$

$\Rightarrow x = 2\sqrt{20-x^2}$

c) $x^2 = 4(20-x^2)$

$5x^2 = 80$

$x = \pm 4$ $(x>0)$

$\Rightarrow x = 4$

Max length $= 2\times 4 + \sqrt{20-4^2}$

$= 10$ units

109. $\underset{\sim}{a}\cdot\underset{\sim}{b} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}\cdot\begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}$

$= 6+-3+-3 = 0$

$\underset{\sim}{a}$ & $\underset{\sim}{b}$ non-zero

$\Rightarrow \underset{\sim}{a} \perp \underset{\sim}{b}$

110 a) $\vec{QP} = \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}$ $\vec{QR} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}$

$\frac{\vec{QP}\cdot\vec{QR}}{|\vec{QP}||\vec{QR}|} = \frac{4}{\sqrt{8}\sqrt{8}} = \frac{1}{2} \Rightarrow \theta = \cos^{-1}\frac{1}{2}$

$= 60°$

b) (i) $M(2,3,2)$

$\underset{\sim}{t} = \frac{1}{3}\underset{\sim}{p} + \frac{2}{3}\underset{\sim}{m} = \begin{pmatrix} 1 \\ 4/3 \\ 0 \end{pmatrix} + \begin{pmatrix} 4/3 \\ 2 \\ 4/3 \end{pmatrix}$

$T(7/3, 10/3, 4/3)$

110 b) ii)

$|\vec{PT}| = \left|\begin{pmatrix} -\frac{2}{3} \\ -2/3 \\ 4/3 \end{pmatrix}\right|$

$\frac{1}{3}\sqrt{24}$

$|\vec{QT}| = \left|\begin{pmatrix} -2/3 \\ 4/3 \\ -2/3 \end{pmatrix}\right|$

$= \frac{1}{3}\sqrt{24}$

$|\vec{RT}| = \left|\begin{pmatrix} 4/3 \\ -2/3 \\ -2/3 \end{pmatrix}\right|$

$= \frac{1}{3}\sqrt{24}$

$\Rightarrow P, Q$ & R equidistant from T.

c) $|\vec{PA}| = \sqrt{3}$
$|\vec{AQ}| = \sqrt{3}$
$|\vec{AR}| = \sqrt{3}$
} P, Q, R equidistant from A

T in same plane as P, Q, R,
A is not,

111. $\vec{PQ} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$

$\vec{QR} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \frac{1}{2}\vec{PQ}$

$\Rightarrow$ QR & PQ parallel
But Q common

$\Rightarrow$ P, Q, R collinear
with Q dividing PR in ratio
2 : 1

112. a) $\vec{BC} = \begin{pmatrix} -4 \\ 2 \\ -3 \end{pmatrix}$

b) $\sqrt{29}$

113. $m_{AB} = \tan\left(\frac{\pi}{2} - \frac{\pi}{3}\right)$

$= \frac{1}{\sqrt{3}}$

114. $a - b + 1 = 0$

$-2a + b + 1 = 0$

$-a \quad + 2 = 0$

$a = 2$

$b = 3.$

115. $\vec{AB} = \vec{DC}$

$\begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -6 - x \\ 4 - y \\ 2 - z \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -11 \\ 2 \\ 3 \end{pmatrix}$

116. $\vec{RS} = \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}$

$\vec{ST} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \frac{1}{3}\vec{RS}$

$\Rightarrow$ ST & RS parallel
But S common

$\Rightarrow$ R, S & T collinear

117. $f'(x) = 2 \times \frac{3}{2} x^{1/2} + 2\sin x \cos x$

$= 3x^{1/2} + 2\sin x \cos x$

118. $(4 - p)^2 + 2^2 + 6^2 = 7^2$

$16 - 8p + p^2 + 4 + 36 = 49$

$p^2 - 8p + 7 = 0$

$(p - 7)(p - 1) = 0$

$p = 7$ or 1

S $(1, 0, 0)$ T $(7, 0, 0)$

119. $\underline{u} + \underline{v} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$ $\underline{u} - \underline{v} = \begin{pmatrix} -4 \\ -2 \\ 4 \end{pmatrix}$

$(\underline{u} + \underline{v}) \cdot (\underline{u} - \underline{v}) = 8 + -16 + 8 = 0$

$\underline{u} + \underline{v}$ & $\underline{u} - \underline{v}$ non-zero

$\Rightarrow \underline{u} + \underline{v}$ & $\underline{u} - \underline{v}$ $\perp$.

120. $\log y = 2\log x + 1$

$\log y = \log x^2 + \log 10$

$\log y = \log 10x^2$

$y = 10x^2$